Operator	$\mathbf{Gate}(\mathbf{s})$		Effec	t (Z)		Effect	(X)	
Pauli-X (X)	- <b>X</b> -		$ 0\rangle \Rightarrow  1\rangle \Rightarrow$	1)  0)		$ +\rangle \rightarrow$ $ -\rangle \rightarrow -$	+>  ->	
Pauli-Z (Z)	$-\mathbf{z}$		$ 0\rangle \Rightarrow  1\rangle \Rightarrow -$	0)  1)		$ +\rangle \rightarrow$ $ -\rangle \rightarrow$	->  +>	
Hadamard (H)	$-\mathbf{H}$	-	$ \begin{array}{c}  0\rangle \rightarrow \\  1\rangle \rightarrow \end{array} $	+>  ->  00>		$ +\rangle \rightarrow$ $ -\rangle \rightarrow$	0)  1)	
Controlled Not (CNOT, CX)			$\begin{array}{c}  00\rangle \Rightarrow \\  01\rangle \Rightarrow \\  10\rangle \Rightarrow \\  11\rangle \Rightarrow \end{array}$	11)  10)  01)				
Controlled Z (CZ)			$\begin{array}{ccc}   00 \rangle & \Rightarrow \\   01 \rangle & \Rightarrow \\   10 \rangle & \Rightarrow \\   11 \rangle & \Rightarrow \end{array}$	00)  01)  10) - 11)				
SWAP			$\begin{array}{ccc} \left  \begin{array}{c} 00 \end{array}\right\rangle & \Rightarrow \\ \left  \begin{array}{c} 01 \end{array}\right\rangle & \Rightarrow \\ \left  \begin{array}{c} 10 \end{array}\right\rangle & \Rightarrow \\ \left  \begin{array}{c} 11 \end{array}\right\rangle & \Rightarrow \end{array}$	00)  10)  01)  11)		$\begin{array}{c}  ++\rangle  \Rightarrow \\  +-\rangle  \Rightarrow \\  -+\rangle  \Rightarrow \\  \rangle  \Rightarrow \end{array}$	++>  -+>  +->  >	
Toffoli (CCNOT, CCX, TOFF)		$\begin{array}{c} -\\  000\rangle \Rightarrow \\  001\rangle \Rightarrow \\  010\rangle \Rightarrow \\  011\rangle \Rightarrow \end{array}$	000)  001)  010)  111)	$\begin{array}{ll} \left  100 \right\rangle & \Rightarrow \\ \left  101 \right\rangle & \Rightarrow \\ \left  110 \right\rangle & \Rightarrow \\ \left  111 \right\rangle & \Rightarrow \end{array}$	100>  101>  110>  011>			

## **Basis Vectors**

$$|0\rangle = \begin{pmatrix} 1\\0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0\\1 \end{pmatrix}$$
$$00\rangle = \begin{pmatrix} 1\\0\\0\\0 \end{pmatrix}, \ |01\rangle = \begin{pmatrix} 0\\1\\0\\0 \end{pmatrix}, \ |10\rangle = \begin{pmatrix} 0\\0\\1\\0 \end{pmatrix}, \ |11\rangle = \begin{pmatrix} 0\\0\\0\\1\\0 \end{pmatrix}$$

## Interference Rules

$$\begin{aligned} |+\rangle &= \sqrt{\frac{1}{2}} |0\rangle + \sqrt{\frac{1}{2}} |1\rangle &\equiv |0\rangle + |1\rangle \\ |-\rangle &= \sqrt{\frac{1}{2}} |0\rangle - \sqrt{\frac{1}{2}} |1\rangle &\equiv |0\rangle - |1\rangle \\ |0\rangle &= \sqrt{\frac{1}{2}} |+\rangle + \sqrt{\frac{1}{2}} |-\rangle &\equiv |+\rangle + |-\rangle \\ |1\rangle &= \sqrt{\frac{1}{2}} |+\rangle - \sqrt{\frac{1}{2}} |-\rangle &\equiv |+\rangle - |-\rangle \end{aligned}$$

## **Dirac Notation**

$$|x\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \quad \Rightarrow \quad |x\rangle = a \,|0\rangle + b \,|1\rangle$$

## **Measurement Rules**

- $\langle 0|0\rangle^2 = 1 \qquad \langle 0|+\rangle^2 = 1/2$  $\langle 0|1\rangle^2 = 0 \qquad \langle 0|-\rangle^2 = 1/2$  $\langle 1|0\rangle^2 = 0 \qquad \langle 1|+\rangle^2 = 1/2$  $\langle 1|1\rangle^2 = 1 \qquad \langle 1|-\rangle^2 = 1/2$
- $\langle +|+\rangle^2 = 1 \qquad \langle +|0\rangle^2 = 1/2$  $\langle +|-\rangle^2 = 0 \qquad \langle +|1\rangle^2 = 1/2$  $\langle -|+\rangle^2 = 0 \qquad \langle -|0\rangle^2 = 1/2$  $\langle -|-\rangle^2 = 1 \qquad \langle -|1\rangle^2 = 1/2$